EE 362: Signals and Systems Name: Solutions

Spring 2019

Homework #3 (Due: Tuesday, February 12, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

1. Consider the N-Point EWMA Filter given by $y[n] = \sum_{i=0}^{N-1} a\left(b^i x[n-i]\right)$ and $a = \frac{1-b}{1-b^N}$, where $0 < b < 1$. When the input $x[n]$ is equal to a constant c for $n \geq 0$, show that the resulting response $y[n] = c$ for all $n \geq N$. [3 points]

2. Compute the unit-pulse response $h[n]$ for $n = 0, 1, 2, 3$ for the following discrete-time systems: [4 points]
(a) $y[n+1] + y[n] = 2x[n]$
(b) $y[n+1] + 0.5y[n] = x[n]$

3. Compute the unit-pulse response $h[n]$ for $n = 0, 1, 2, 3$ for the following discrete-time system: [3 points]

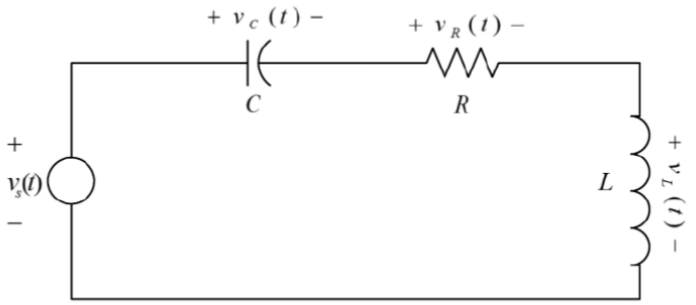
$y[n+1] + 0.5y[n] = x[n+1]$

4. For the discrete-time signals $x[n]$ and $v[n]$ given below, compute the convolution $x[n] * v[n]$. [4 points]
$x[0] = 4; x[1] = 1; x[3] = -1; x[n] = 0$ for all other integers n;
$v[0] = 1; v[1] = -2; v[2] = 3;\ v[3] = -4; v[n] = 0$ for all other integers n.

5. Find a closed-form solution for the unit-pulse response of the following system. [4 points]

$$y[n+1] + 1.5y[n] = x[n]$$

6. For the RLC circuit shown below, find the input/output differential equation when the output is the voltage across the capacitor. [4 points]



(1) 2.1

$$y[n] = \sum_{i=0}^{N-1} a b^i x[n-i] \quad \text{and} \quad a = \frac{1-b}{1-b^N}$$

$x[n] = c$

$$\therefore y[n] = ab^0 x[n] + ab^1 x[n-1] + ab^2 x[n-2] + \cdots + ab^{N-1} x[n-N+1]$$
$$= ab^0 c + ab^1 c + ab^2 c + \cdots + ab^{N-1} c$$
$$= a(1 + b + b^2 + \cdots + b^{N-1})c$$
$$= \frac{1-b}{1-b^N}(1 + b + b^2 + \cdots + b^{N-1})c$$
$$= \frac{c}{1-b^N}\left[1 + b + b^2 + b^3 + b^4 + \cdots + b^{N-1} - b - b^2 - b^3 - b^4 - \cdots - b^{N-1} - b^N\right]$$
$$= \frac{c}{1-b^N}[1 - b^N] = c$$

We could have arrived to same solution by realizing that- $\sum_{i=0}^{N-1} ab^i = 1$

(2) 2.5

$$y[n+1] = -y[n] + 2x[n]$$

(a) for unit-pulse response set $x[n] = \delta[n]$ and label $y[n]$ as $h[n]$

$$\therefore h[n+1] = -h[n] + 2\delta[n]$$

for $h[0]$, $n = -1$

$$\therefore h[0] = -h[-1] + 2\delta[-1]$$

- The unit-pulse response is zero for $n < 0$

$\delta[n] = 1$ only when $n = 0$

- $n = -1 \rightarrow h[0] = -h[-1] + 2\delta[-1] = 0 + 2 \times 0 = 0$
- $n = 0 \rightarrow h[1] = -h[0] + 2\delta[0] = 0 + 2 \times 1 = 2$
- $n = 1 \rightarrow h[2] = -h[1] + 2\delta[1] = -2 + 0 = -2$
- $n = 2 \rightarrow h[3] = -h[2] + 2\delta[2] = -(-2) + 0 = 2$

(b)

$$y[n+1] + \tfrac{1}{2} y[n] = x[n]$$

$$\therefore h[n+1] = -\tfrac{1}{2} h[n] + \delta[n]$$

$h[n]$ for $n = -1$

- $n = -1 \rightarrow h[0] = -\tfrac{1}{2} h[-1] + \delta[-1] = 0 + 0 = 0$
- $n = 0 \rightarrow h[1] = -\tfrac{1}{2} h[0] + \delta[0] = 0 + 1 = 1$
- $n = 1 \rightarrow h[2] = -\tfrac{1}{2} h[1] + \delta[1] = -\tfrac{1}{2}(1) + 0 = -\tfrac{1}{2}$
- $n = 2 \rightarrow h[3] = -\tfrac{1}{2} h[2] + \delta[2] = -\tfrac{1}{2}(-\tfrac{1}{2}) + 0 = \tfrac{1}{4}$

(3) 2.6

(b) $y[n+1] + \frac{1}{2}y[n] = x[n+1]$

$\therefore y[n+1] = -\frac{1}{2}y[n] + x[n+1]$

$\therefore h[n+1] = -\frac{1}{2}h[n] + \delta[n+1]$

$n=-1 \rightarrow h[0] = -\frac{1}{2}h[-1] + \delta[0] = 0 + 1 = 1$
$n=0 \rightarrow h[1] = -\frac{1}{2}h[0] + \delta[1] = -\frac{1}{2}(1) + 0 = -\frac{1}{2}$
$n=1 \rightarrow h[2] = -\frac{1}{2}h[1] + \delta[2] = -\frac{1}{2}(-\frac{1}{2}) + 0 = \frac{1}{4}$
$n=2 \rightarrow h[3] = -\frac{1}{2}h[2] + \delta[3] = -\frac{1}{2}(\frac{1}{4}) + 0 = -\frac{1}{8}$

and so on

$\therefore h[n] = (-1)^n (\frac{1}{2})^n \quad , n \geq 0$

or $h[n] = (-\frac{1}{2})^n u[n]$

(4) 2.8(a)

$P = 0$ and $Q = 0$

$y[P+Q+1] = ?$

$y[0] = 4$
$y[1] = -7$
$y[2] = 10$
$y[3] = 14$
$y[4] = -2$
$y[5] = -3$
$y[6] = 4$

	$x[n]$			
	4	1	0	−1
1	4	1	0	−1
−2	−8	−2	0	2
$v[n]$ 3	12	3	0	−3
−4	−16	−4	0	4

(5)

$y[n+1] = -1.5y[n] + x[n]$

$h[n+1] = -1.5h[n] + \delta[n]$

$h[0] = -1.5h[-1] + \delta[-1] = 0 + 0 = 0$

$h[1] = -1.5h[0] + \delta[0] = 1$

$h[2] = -1.5h[1] + \delta[1] = -1.5$

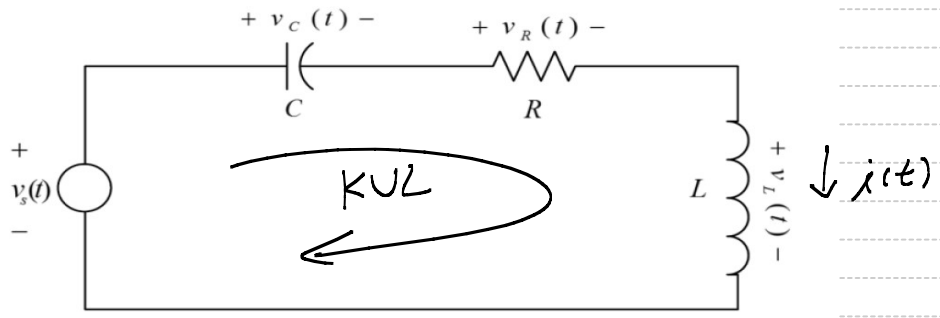
$h[3] = -1.5h[2] + \delta[2] = (-1.5)^2$

$\therefore h[n] = (-1.5)^{n-1}$ for $n \geq 1$
$= 0$ for other values of n

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$x(t) = v_s(t)$

$y(t) = v_c(t)$



$$-v_s(t) + v_c(t) + v_R(t) + v_L(t) = 0$$

$$-x(t) + y(t) + R\,i(t) + L\frac{di(t)}{dt} = 0$$

$$i(t) = i_c(t) = C\frac{dv_c(t)}{dt} = C\frac{dy(t)}{dt}$$

$$\therefore -x(t) + y(t) + RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} = 0$$

OR

$$\frac{d^2y(t)}{dt^2} + \frac{R}{L}\frac{dy(t)}{dt} + \frac{1}{LC}y(t) = \frac{1}{LC}x(t)$$